Recurrence plot analysis of nonstationary data: The understanding of curved patterns

A. Facchini,[1,*] H. Kantz,[2,†] and E. Tiezzi[1,‡]
[1]*Dept. of Chemical and Biosystems Sciences, Siena University, Siena, Italy*
[2]*Max Planck Institute for the Physics of Complex Systems, Dresden, Germany*
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Recurrence plots of the calls of the *Nomascus concolor* (Western black crested gibbon) and *Hylobates lar* (White-handed gibbon) show characteristic circular, curved, and hyperbolic patterns superimposed to the main temporal scale of the signal. It is shown that these patterns are related to particular nonstationarities in the signal. Some of them can be reproduced by artificial signals like frequency modulated sinusoids and sinusoids with time divergent frequency. These modulations are too faint to be resolved by conventional time-frequency analysis with similar precision. Therefore, recurrence plots act as a magnifying glass for the detection of multiple temporal scales in slightly modulated signals. The detected phenomena in these acoustic signals can be explained in the biomechanical context by taking in account the role of the muscles controlling the vocal folds.

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I. MAMMAL VOCALIZATIONS

The sound production of living beings is usually due to rather basic physical mechanisms. In mammals the vocal folds, together with glottal airflow, constitute a highly nonlinear oscillator. The vocal folds, according to the known myoelastic modelling theory [1] are set in oscillation by a combined effect of lung pressure (Bernoulli effect) and by the control of the muscles of the vocal apparatus controlling the motion of the vocal folds. The characteristic frequency of the vocal pitch is determined by muscle tension and action, while the vocal tract acts as a filter for the production of the vocalization [2]. Normal sustained vocalizations seem to be completely periodic signals, but small irregularities and perturbations add to voice the naturalness that we know. Under certain conditions, much larger irregularities arise in sustained vocalizations, adding very irregular components [3]. In recent years, several studies have investigated these phenomena with techniques borrowed from nonlinear time series analysis and nonlinear dynamical systems. In particular, in human and animal voice, transition phenomena from periodic to aperiodic and chaotic behaviors such as bifurcations, period doubling, and toroidal oscillations were observed [4–6].

Using Poincaré sections, estimation of attractor dimension and computation of Lyapunov exponents [7], these irregularities can be related to low-dimensional chaotic phenomena, and can be reproduced by theoretical models [8,9]. For example, the two mass model (the most accepted one for the mammal apparatus of phonation) can exhibit irregular oscillations [10–12]. The apparatus of phonation can be investigated through the characterization of the animal vocalization, where vocal nonlinearity can be used. The nonlinear analysis of human speech signals has been carried out extensively, while nonlinear characteristics for animal voice signals have not yet been investigated [13].

II. TIME SERIES ANALYSIS BASED ON RECURRENCE PLOTS

Recurrent behaviors are typical of natural systems. In the framework of dynamical systems, this implies the recurrence of state vectors, i.e., states with large temporal distances may be close in state space. The recurrence plot, proposed by Eckmann *et al.* [14], is a visual tool able to identify spatiotemporal recurrences in multidimensional phase spaces. Since phase spaces of more than two dimensions can be only visualized by a projection, it is hard to investigate recurrences directly, i.e., without a statistical tool. Starting from the time series $s(t)=\{s_1,\ldots,s_n\}$, the attractor of the underlying dynamics is reconstructed in a phase space by applying the time delay vector method by Takens and Mañe [15]. The reconstructed trajectory $\mathbf{X}$ can be expressed as a matrix where each row is a phase space vector:

$$\mathbf{X}=[x_1,x_2,\ldots,x_m]^T, \tag{1}$$

where $x_i=[s_i,s_{i+T},\ldots,s_{i+(D_E-1)T}]$ and $m=n-(D_E-1)T$, where D_E is the *embedding dimension* and T is the *delay time*.

The *recurrence plot* (RP) is a tool able to investigate higher dimensional dynamics through a two dimensional binary plot of its recurrences. Any recurrence of state i with state j is pictured on a boolean matrix expressed by [16]:

$$\mathbf{R}_{i,j}^{D_E,\epsilon}=\Theta(\epsilon-\|x_i-x_j\|), \tag{2}$$

where $x_{i,j}\in\mathbb{R}^{D_E}$ are the embedded vectors, $i,j\in\mathbb{N},\Theta(\cdot)$ is the Heaviside step function and ϵ is an arbitrary threshold. In the graphical representation, each nonzero entry of $R_{i,j}$ is marked by a black dot in the position (i,j). Since any state is recurrent with itself, the RP matrix fulfills $R_{i,i}=1$ which hence contains the diagonal *line of identity* (LOI).

To compute a RP, the norm in Eq. (2) must be defined. We use the L_∞ norm, because it is independent of the phase space

*Electronic address: a.facchini@unisi.it
†Electronic address: kantz@mpipks-dresden.mpg.de
‡Electronic address: tiezzienzo@unisi.it

dimension and no rescaling of ϵ is required. Furthermore, special attention must be given to the choice of the threshold ϵ. There is not a specific guideline for this estimation, but the noise level of the time series must be taken into account. Values suggested in the literature are some percentage of the maximum diameter of the attractor (in any case, not more than 10%). RPs are widely used in searching for deterministic dynamics in highly irregular *stationary* time series, since the characteristic textures of deterministic behaviors are distributions of short lines parallel to the LOI. The *recurrence quantification analysis* [17] is mainly based on statistical descriptions of these parallel line patterns, and it is useful to analyze time series with high levels of noise.

Recently, RPs were used in the analysis of *nonstationary* time series. In this case, the traditional methods of time series analysis are not adequate for the computation of the characteristic parameters that identify chaotic dynamics such as Lyapunov exponents, correlation dimension, and embedding dimension [18], and some papers are mainly concentrated on the origin of blocklike or squarelike patterns [19] or in line structures [20].

Since voice is a strongly nonstationary process, RP in the past years were used to analyze large time series of human and animal vocalizations [21]. In [16] RPs were used to identify driving forces in nonautonomous systems, which, however, does not easily translate to the voice analysis.

III. EXPERIMENTAL AND ANALYSIS METHODS

We consider calls vocalized by gibbons *Nomascus concolor*, and *Hylobates lar* [26]. The motivation for the study of these particular signals comes from the fact that such calls are used in gibbons taxonomy [22]. We search for a techinque able to identify a *vocal fingerprint* of the animal. Once available, this would constitute an important improvement of the methodology used to classify Gibbon species. The calls, technically called "Notes," are parts of a larger vocalization structure known as "Song." A song is what fullfils the criteria set up by Thorpe [23]: "*What is usually understood by the term song is a series of notes, generally more than one type, uttered in succession and so related as to form a recognizable sequence of pattern in time*." Within a gibbon song, a note is any single continuous sound of any distinct frequency or frequency modulation which may be produced during either inhalation or exhalation [24,25]. Every call is recorded at a sampling frequency of 11025 Hz and preliminary analysis is performed by spectrograms (1024 points FFT). As can be seen in Figs. 1 and 2 the spectrograms of the vocalization are typical of periodic signals. For the recurrence analysis, we use time delay embeddings with an embedding dimension $D_E=8$ and a delay time $T_d=3$ for all the signals, while the RPs were computed considering the maximum norm with a fixed neighborhood size threshold ϵ. The embedding dimension was chosen under consideration of the dimensionality of the two mass model ($D_E=4$), while the delay time was chosen in correspondence of the first minimum of the average time delayed mutual information. The results from the spectrogram-based analysis reveal that the signals are almost periodic for the entire considered window, and are characterized by slight amplitude modulation. Only in the call of the *Hylobates lar* slight frequency modulation can be noticed.

IV. OBSERVED PATTERNS IN NATURAL DATA

All the recurrence plots are characterized by white and textured rectangular regions. The white ones, corresponding to zones in which recurrences do not occur, are essentially caused by the corresponding time series segments having amplitudes which differ by more than ϵ. As it will be outlined later, every slow amplitude variation causes such white areas. Our attention is focused on the nature of the patterns observed in the textured zones, formed by aggregation of short straight lines in curves of different shapes.

On the very small scales, every nonempty part of the recurrence plot is composed of lines parallel to the diagonal, with a spacing of about 11 sample points (magnified in Figs. 3 and 4). These lines are trivial recurrences of a periodic signal and reflect the pitch of the sound (the distance of 11 sample points corresponds to a frequency of about 1000 Hz). We hence call these lines fundamental lines in the following. The fact that these fundamental lines are interrupted in a way that on larger scales regular patterns appear is the nontrivial aspect which we study in the following.

In the RP of *Nomascus concolor* (Fig. 1) these regular patterns of fundamental lines form hyperbolic structures convergent to the LOI.

The call of *Hylobates lar* (Fig. 2) presents some slight oscillations in the frequency, and in its RP two main patterns are present: some curved aggregates and "circular concentric gaps." The last are the most interesting, since they are not formed by aggregates of lines, but by gaps forming concentric circles in zones where the pattern is usually referred to periodic signals.

V. OBSERVED PATTERNS IN ARTIFICIAL DATA

First of all, in order to reproduce the white blocks observed in the RPs of the vocalizations, we consider a sinusoidal signal with a given carrier frequency and with a piecewise constant amplitude. Evidently, if i and j refer to delay vectors of segments of the same amplitude, then a recurrence is found if the difference $|i-j|$ is a multiple of the period of the carrier. This creates lines parallel to the LOI with mutual spacings identical to the number of sample points per period. If i and j stem from data segments with different amplitudes, whose difference is larger than ϵ, recurrences cannot occur. Hence, such a signal with a stepwise amplitude modulation creates rectangular blocks of recurrences and nonrecurrences.

Since in gibbon vocalizations frequency modulation phenomena are to be expected (and can be partly seen in the periodograms), we try to trace back the nontrivial patterns observed in the RPs considering signals with a time dependent frequency. In view of the preceeding analysis we consider sinusoidal signals sampled at 11025 Hz and with a fixed carrier of 1000 Hz as a good approximation of the ex-

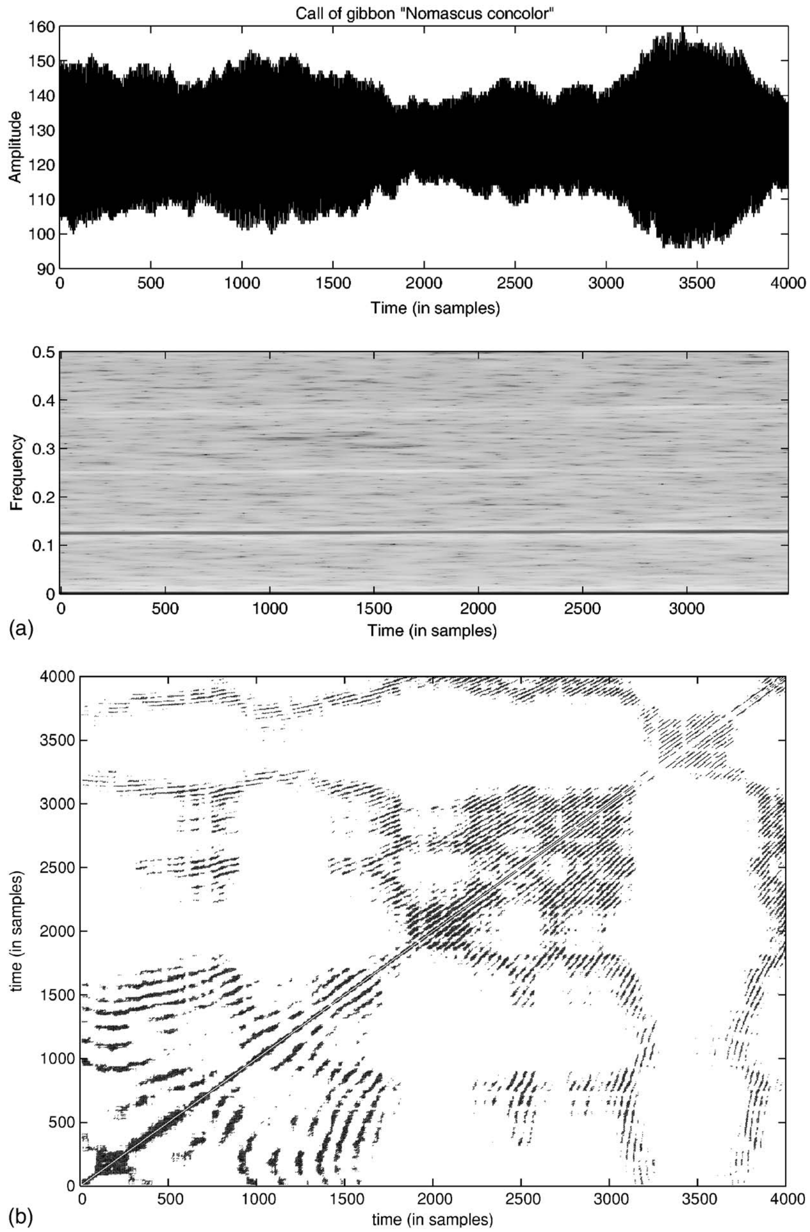

FIG. 1. Analysis of the call of a *Nomascus concolor* gibbon. In the time domain the signal shows some amplitude modulation, while the spectrogram exhibits a line corresponding to one isolated almost constant frequency without pronounced harmonics. The inspection of the RP shows white and textured boxes. The white ones are due to amplitude variations in the signal, in textured ones the existence of two times scales is evident, one related to the carrier frequency, the other formed by hyperbolic macropatterns converging toward the LOI.

perimental data. We then impose two types of frequency variation.

(1) $\cos(2\pi 1000t+\alpha\pi t^2)$: A linear increase of the carrier frequency, controlled by the parameter α.

(2) $\cos(2\pi 800t+\beta\sin(2\pi f_m t))$: a periodic frequency modulation around a fixed carrier, where the modulation is characterized by the amplitude β and the frequency f_m.

In Fig. 3 the typical RP of the signal $\cos(2\pi 1000t+\alpha\pi t^2)$ is shown. The patterns are of the same type of those observed in the *Nomascus concolor* gibbon (Fig. 1). Apart from the evident carrier frequency, the only free parameter is the frequency increase α. We find that $\alpha\in[70,130]$ generates patterns similar to the natural ones. On the considered time series length of 4000 samples (0.36 seconds) this corresponds to frequency shifts in between 28 Hz and 52 Hz, comparing the initial frequency to the final one, $1000+\alpha t$. The signal in the figure was produced using $\alpha=100$, with a frequency shift of 40 Hz, which is very difficult to identify by a spectrogram inspection (compare Fig. 1).

The characteristic parameters of the second signal are β and f_m, which plays an important role in the shape of the RPs that can be obtained. The pattern of the *Hylobates lar* gibbon can be accurately reproduced using $\beta=10$ and $f_m=8$ (Fig. 5), with a frequency shift of ±80 Hz. It should also be noticed here that the white circular regions in the recurrence plot are not due to amplitude variations, but to frequency variations in the signal which produce a periodic shift in the reconstructed trajectories of the attractor. One cycle of the periodic frequency modulation contains about 1400 sample points. In fact, this modulation can be observed in Fig. 2. Whereas the cycle length of the frequency modulation can be obtained quite accurately from the periodogram, the precise amplitude of the frequency variation is less clear. This can be obtained quite accurately from the recurrence plot, where the parameter β is translated into the width of the nonempty stipes. The above value $\beta=10$ was found by matching their width with the observed patterns in Fig. 2.

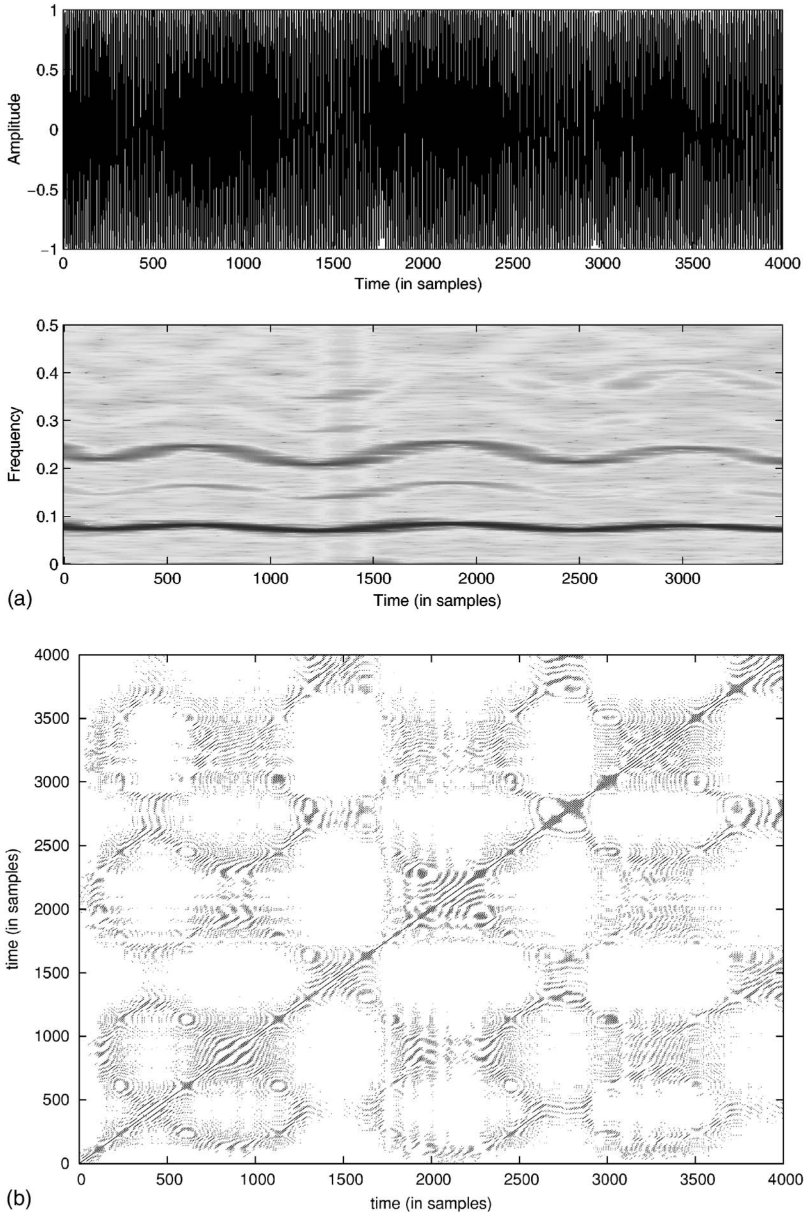

FIG. 2. Analysis of the call of *Hylobates lar*. In the spectrogram a slight frequency modulation is visible. The strong harmonics are related to the non-sinusoidal nature of the signal which is artificially imposed by the cutoff of the amplitudes by the recording. In the RP two interesting patterns can be identified: curved and concentric gapped patterns appear periodically in the RP, while white circular regions are due to the frequency shift inside the signal.

VI. ORIGIN OF THE PATTERNS

Only a signal continuous in time has perfect recurrences. The same signal recorded with a discrete sampling time generally will have imperfect recurrences, since the exact ones may be omitted by the sampling. For a sinusoidal signal, ideally the RP should contain all lines parallel to the LOI whose distances to the diagonal are integer multiple of the period. Instead, due to the phase error introduced by the sampling, several of these lines result broken or absent. An increase of the ϵ will join the broken lines, but the others have the tendency to increase their width. In this sense, the threshold ϵ represents the tolerable phase mismatch between two delay vectors in order to be still neighbors in the sense of $|\vec{x}_i - \vec{x}_j| < \epsilon$. A recurrence happens when an integer multiple of the sampling interval matches another integer multiple of the period so that their difference modulo 2π is less than the phase tolerance.

The macroscopic structures observed are a result of the interplay between signal and sampling. At a constant sampling rate they are typical of the signal, since the temporal variation of the frequency introduces further changes both in the position of the samples and in the position of the points in the reconstructed phase space. For nonmodulated signals this can be easily shown. The discrete time series $s_j = \cos(2\pi f / f_s j)$ has *perfect* recurrences when

$$2\pi \frac{f}{f_s} j = 2k\pi \Rightarrow j = \frac{f_s}{f} k \quad j,k \in \mathbb{N}, \tag{3}$$

where f_s is the sampling frequency. Let us first assume that f_s/f is an integer. Then for $k=1$, j is the number of samples contained in a period. In the reconstructed phase space the state of the system repeats itself every f_s/f samples and a recurrence between the sample j_k and j_{k+1} occurs. Since in general the ratio f_s/f is not integer, generally a phase error is introduced and the position of the state vector at j_{k+1} is slightly different from the one at j_k, related to the noninteger part of the ratio f_s/f. If this difference is less than ϵ, the

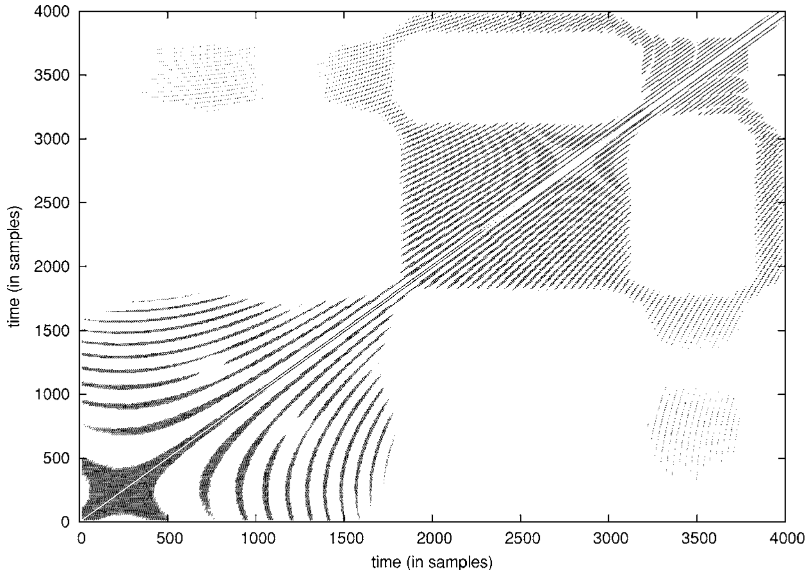

FIG. 3. Reproduction of the patterns observed in the vocalization of *Nomascus concolor* by the signal $\cos(2\pi 1000t+100\pi t^2)$. The RP was obtained using $(D_E=3, T=3, \epsilon=0.08)$. From the inspection two different scales emerge: the lower is relative to the macrostructures which build the periodic hyperbolic patterns, the higher is relative to the aggregation of short parallel lines building the macrolines. The signal has a frequency shift of 40 Hz.

recurrence is nonetheless identified. However, after a certain amount of time shift t^* measured in terms of k, the recurrence between the state vectors $\vec{x}_k$ and $\vec{x}_{k^*}$ is missed. A new recurrence will happen only when, for a certain k_M the corresponding j_M approaches again an integer. Therefore, the phase shift introduces a second, larger macrofrequency f_M that depends on the ratio f_s/f. For rational $f_s/f=q+r/s$, $0<r<s, q,r,s \in \mathbb{N}$, this frequency is f/s. For irrational f_s/f, such a frequency does not exist and the macropattern is quasiperiodic. In both cases, macrostructure in RPs of perfectly periodic signals are therefore large macrolines parallel to the LOI, with a time period of t_M, and composed by parallel lines spaced with time $1/f$. This thinking can be easily extended to modulated signals. In this case the value of the macrofrequency changes in time and the patterns will be more complex. In particular, for the linearly modulated signal, the frequency of the macropatterns evidently increases in time.

VII. CONCLUSIONS

Recurrence plots are sensitive to the change of signal properties in the course of time. Since no windowing is involved, they are very good candidates for the highly time resolved analysis of nonstationary signals, as it has been observed in the literature before. However, the crucial point is that the information contained in the potentially very complex patterns of such data representations need a suitable interpretation.

Our analysis of the calls from gibbons of two species shows that in natural signals certain types of nonstationarities exist and can be identified by RP analysis with much better precision than with periodograms. Our signals are characterized in the frequency domain by seemingly strictly periodic behaviors, with nonstationarities only visible in the amplitude of the signal. Despite these observations, ethologistic literature states that these calls are characterized by

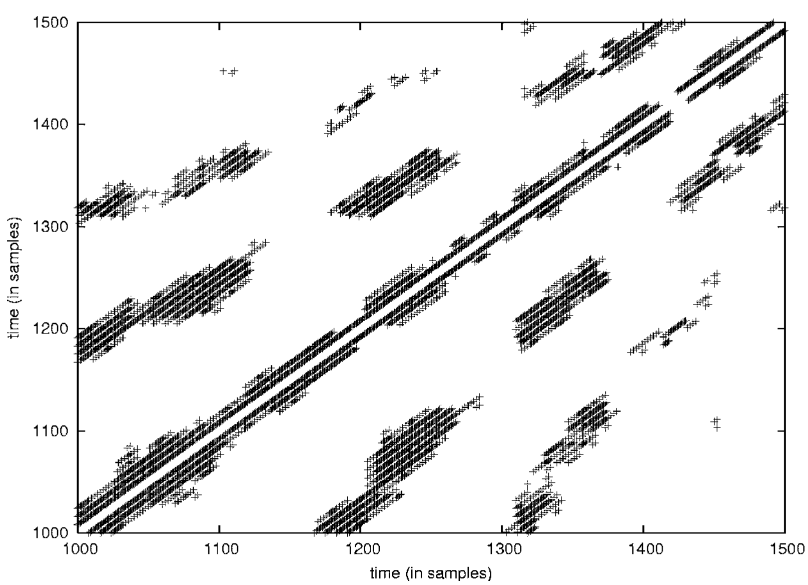

FIG. 4. A magnified zone of the RP of the *Nomascus concolor* gibbon. The parallel lines forming the macropatterns have a spatial period of 11 time samples, related to the pitch of the signal.

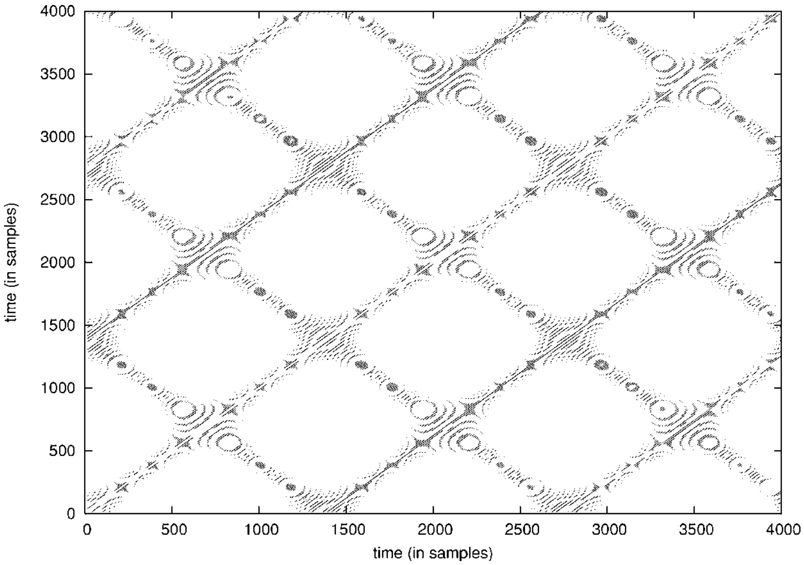

FIG. 5. Reproduction of "gapped" patterns observed in the call of *Hylobates lar* by the signal $\cos(2\pi 800t+10\sin(2\pi 8t))$. The RP was computed using $D_E=3$, $T=3$, and $\epsilon=0.05$.

some kind of frequency modulation, which varies from one species to another. The recurrence plot analysis performed on the calls put in evidence some long-time nonstationarities which can be individuated by the formation of curved and "gapped" patterns formed by aggregation of short lines parallel to the LOI, forming circular and hyperbolic lines. These very characteristic patterns could be interpreted by either linearly increasing carrier frequencies or by periodically modulated carrier frequencies, where in both cases the frequency changes are too small for a proper characterization on the basis of spectrograms. In this sense, recurrence plot analysis acts like a magnifying glass for these effects, and the reason is of course that we observe some kind of interference effect between sampling time and the continuous signal, which influences the position of the state vectors in the reconstructed phase space, determining a lack of recurrences in the recurrence plot.

Future work will consist in a better understanding and characterization of these phenomena, taking in consideration a larger class of signals and physical phenomena.

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